CSCI 3310 Homework 4

1. In a survey, 30 students reported whether they like potatoes Mashed, French fried, or Twice-baked. 15 like mashed, 20 like French fries, and 9 like twice-baked potatoes. Additionally, 12 students like both mashed and fried potatoes, 5 like French fries and twice-baked potatoes, 6 like mashed and twice-baked, and 3 like all three styles. How many students hate potatoes? Explain why your answer is correct.

2. Let $A$, $B$, and $C$ be sets:
   (a) Find $|(A \cup C) \setminus B|$ provided $|A| = 50$, $|B| = 45$, $|C| = 40$, $|A \cap B| = 20$, $|A \cap C| = 15$, $|B \cap C| = 23$, and $|A \cap B \cap C| = 12$.
   (b) Describe a set in terms of $A$, $B$, and $C$ with cardinality 26.

3. Answer the following questions over length 7 numbers (strings over $\Sigma = \{0, 1, \ldots, 9\}$, thus 6 = “0000006”):
   (a) How many possible strings are there?
   (b) How many are palindromes?
   (c) How many are palindromes with a ‘7’ in the middle?
   (d) How many have “333” as a substring?

4. Answer the same questions of Problem 3 if we only consider strings of numbers without leading 0’s?

5. Answer the following questions for the set $S = \{1, 2, \ldots, 16\}$:
   (a) How many size-11 subsets does $S$ have?
   (b) How many size-11 subsets of $S$ contain a 3?
   (c) How many size-11 subsets of $S$ contain at least one odd number?
   (d) How many size-11 subsets of $S$ contain exactly one even number?

6. How many length-6 strings over $\Sigma = \{0, 1, 2\}$ have exactly two 2’s?

7. Expand $(x + y)^6$ using binomial coefficients.

8. Using the digits $\{2, 3, \ldots, 8\}$, find the number of different 5-digit numbers such that:
   (a) Digits can be used more than once.
   (b) Digits cannot be repeated, but can come in any order.
   (c) Digits cannot be repeated and must be written in increasing order.
   (d) Which of the above counting questions is a combination and which is a permutation? Explain why this makes sense.

9. An anagram of a word is just a rearrangement of its letters. How many different anagrams of “uncopyrightable” are there? (This happens to be the longest common English word without any repeated letters.)

10. Answer the following given I have 27 arrows and a bow and 10 targets to shoot at:
    (a) How many ways are there to distribute the arrows to the targets?
    (b) How many ways are there to distribute the arrows to the targets ensuring at least one arrow per target (obviously a bullseye)?
    (c) How many ways are there to shoot the targets such that every target has at least 2 arrows, and one target has only 1?