

CSCI 4341

Assignment 1 (100 points)

The homework is to be turned in by 5 P.M. on the day it is due. Style and correctness will be graded – be neat and thoroughly explain each step. It is expected that you will work with your groups to help each other understand the questions and answers. Anything that is not clear will be counted wrong. Problems 3, 4, 5 must also be explained orally by appointment.

Problem 1 (10): Explain why **NP** does not stand for *non-polynomial* (even if $\mathbf{P} \neq \mathbf{NP}$ or $\mathbf{P} = \mathbf{NP}$).

Problem 2 (15): Give a polynomial time algorithm to decide *2SAT*, i.e., find an assignment of the variables that show it is satisfiable, or that it can not be satisfied.

Problem 3 (25): Show that the Traveling Salesman Problem is **NP**-complete with the following two steps:

1. Show that the problem is in **NP**.
2. Show that *Undirected Hamiltonian Path* is poly-time reducible to the problem.

Problem 4 (25): Given an undirected graph G . $SPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at most } k \text{ from } a \text{ to } b \}$. Prove that $SPATH \in \mathbf{P}$.

Problem 5 (25): Given an undirected graph G . $LPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at least } k \text{ from } a \text{ to } b \}$. Prove that $LPATH$ is **NP**-complete with a reduction from Hamiltonian Path, $UHAMPATH = \{ \langle G, s, t \rangle \mid G \text{ is an undirected graph with a Hamiltonian path from } s \text{ to } t \}$.

Bonus (12): Tetrominos are four squares stuck together along an edge (Tetris pieces). There are five distinct tetromino types: the straight, square, L-shaped, T-shaped, and Z-shaped tetromino. Is it possible to tile (i.e., cover exactly without overlaps) an 8×8 chessboard with the following? If possible, give the tiling. If not, explain why.

- (a) 16 straight tetrominoes
- (b) 16 square tetrominoes
- (c) 16 L-tetrominoes
- (d) 16 T-tetrominoes
- (e) 16 Z-tetrominoes
- (f) 15 T-tetrominoes and one square tetromino