# Games, Puzzles & Computation

Notes: Week 3/20/2017 - 3/24/2017

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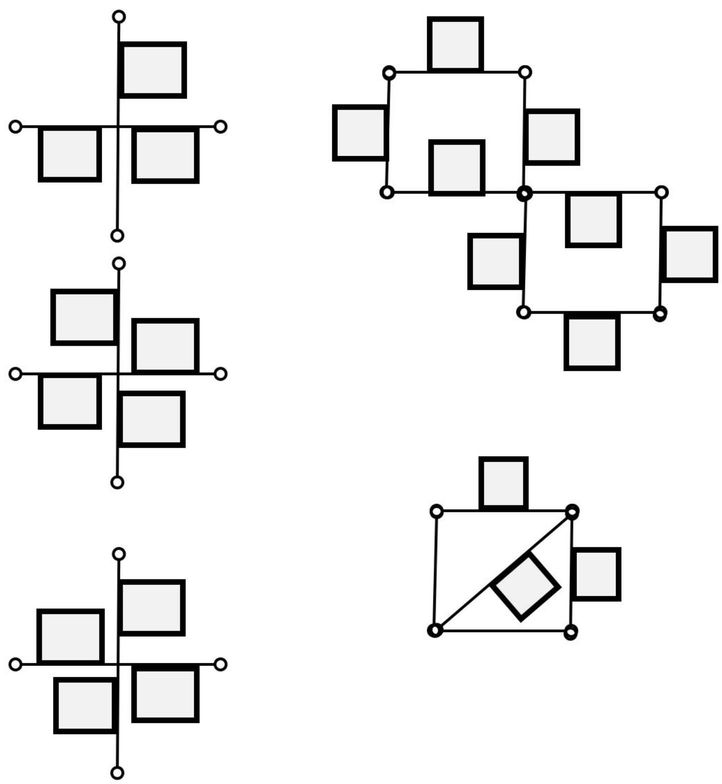
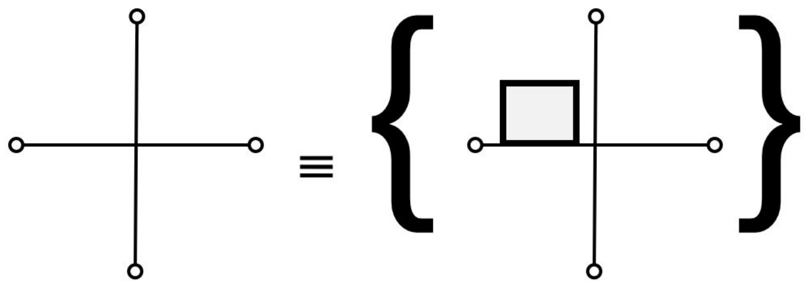
Game: Hedging Edges

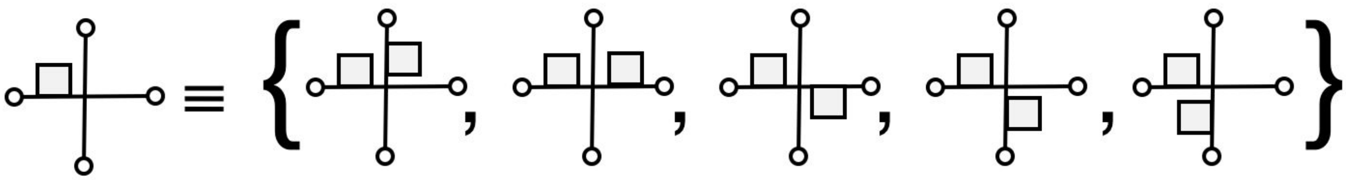
- The board is a planar graph
- The move is labeling an edge with a unit square label (one label per edge)
- Examples:



- Impartial game
  - Every position has a value

 $\equiv \{\_\} \equiv \{*0\} \equiv *1$


≡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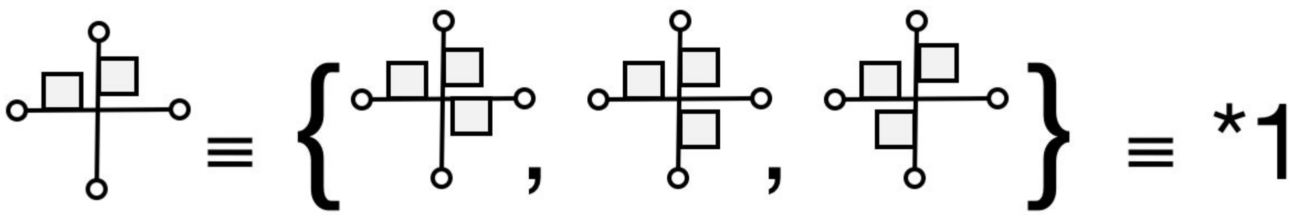
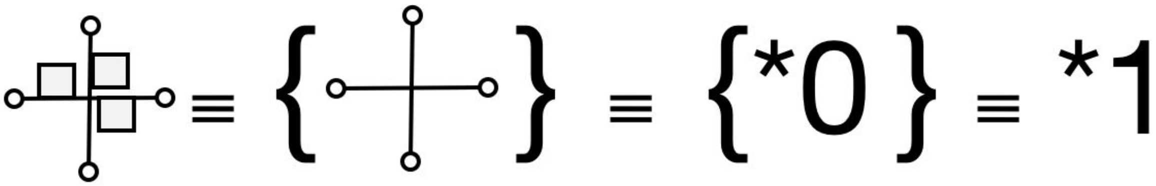

≡
≡ *1

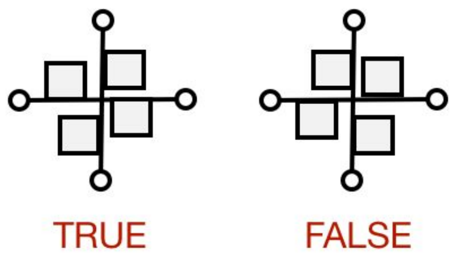

≡
≡ {*0} ≡ *1

Complexity

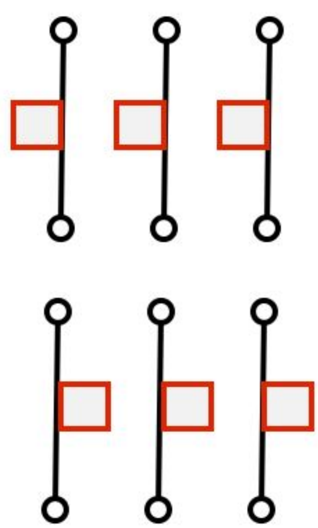
- Question:
    - Given a position in the Hedging Edges game, is it possible to label all the remaining edges?

Reduction: reduce form planar 3-SAT

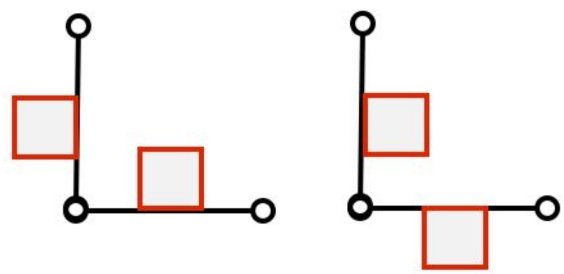
- Variable Gadget



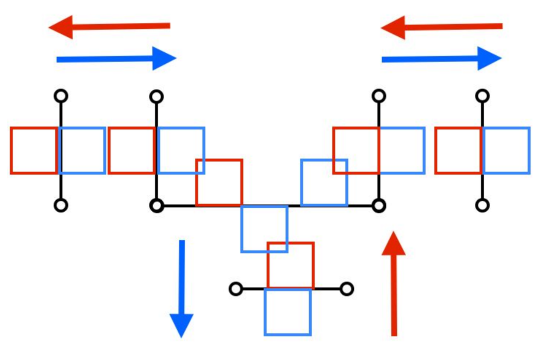

- Propagation gadget: move info from one part to the other



- Turning Gadget



- Clause Gadget



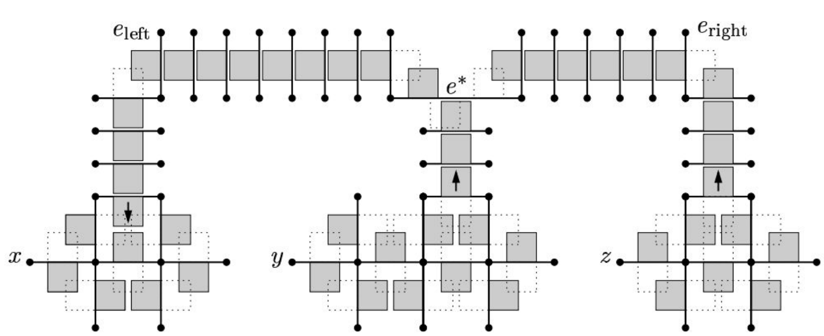



Figure 2: Encoding the clause of a Boolean formula; here $\overline{x} \vee y \vee \overline{z}$

Moving to QBF

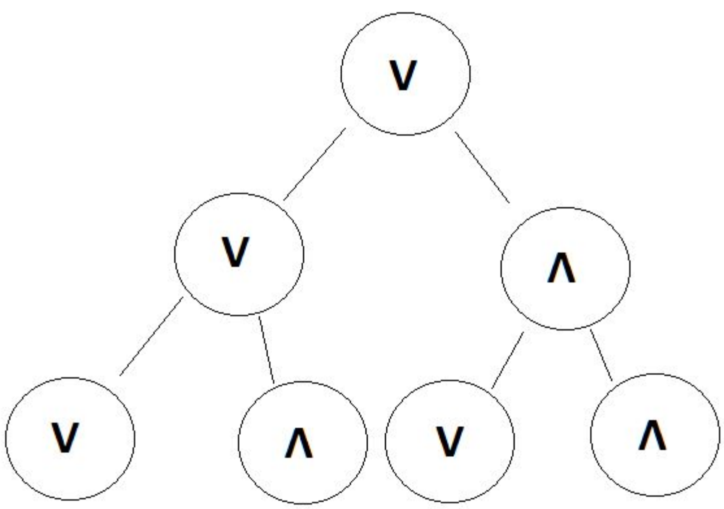
What do we know?
P ⊆ NP ⊆ PSPACE ⊆ EXPTIME ⊆ NEXPTIME ⊆ EXPSPACE
NPSPACE = PSPACE
P ⊊ EXPTIME

QBF: Quantified Boolean Formula

- Basically just SAT as a game
- For Sat $\phi(X_1, X_2, \ldots X_n) = (X_1 \vee \neg X_3 \vee X_4 \vee X_2)\ \forall\ X_1$
    - NP, the certificate is a setting of variables (there exist a setting)
    - coNP (complement of NP), that no setting of the variables make $\phi$ true. For all settings)

Alternation

- A non-deterministic computation accepts if any one of the branches accepts
- equivalent to an "or" function
- Alternating Computation
    - Nodes can be designated "or" or "and" when a split happens
- Nodes can be "and"
    - all end states must accept from split of "and" vertex



The Polynomial Hierarchy

- Alternating computation yields a natural hierarchy of classes within PSPACE
- Σi - i levels of alternation ∃ ∀ ∃ ∀…
- ∏i - i levels but starting with ∀: ∀ ∃ ∀ ∃…

- How do we show that something is PSPACE-complete?

PSPACE-Complete

1.Show ∈ PSPACE

2. Every A in PSPACE is polynomial time reducible to our problem

Reduce PSPACE-complete problem to your problem

Fully quantified - every variable appears with a quantifier.

TQBF={<Ø> | Ø is true fully quantified Boolean formula}

Thm. TBQF is PSPACE-Complete.

The Formula Game

- Artificial game based on TQBF.
- Given Ø = $\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5 \ldots Qx_k[\Psi]$
- Q is either ∃, or ∀.
- Ψ is some boolean formula with x1...xk

- Player A selects x1 is T/F
- Player B selects x2 is T/F
- After all k variables are set:
  A wins if Ψ is true, B wins if Ψ is false.

- Ex.
  ∃x1∀x2∃x3 [(x1 ∨ x2) ^ (x2 ∨ x3) ^ (x2 ∨ x3)]

  P1 wins if both play optimally.