Group K - Notes

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Abstract

The notes for the following document are based on Algorithmic Problems in Tiling.

1 Introduction

What is tiling? Tiling is a representation of a geometric shape that is a copy of the same shape though out the whole plane. The question that is asked in tiling is, "Is there an algorithm that will find or do a tiling plane? ". The answer is no, there is no algorithm yet. Tiling can either be isohedral or non-isohedral.

2 History - Are there anisohedral with plane tiling?

The study of plane tiling in anisohedral shapes begins with the famous German mathematician, David Hilbert. Hilbert's problems were twenty-three problems in mathematics published by him in 1900. The basis for the eighteenth is that he assumed there are no anisohedral shapes.

However, in 1935, Heinrich Heesch gave an example of an anisohedral tile which cannot tile the plane with only one symmetry class.

Later in 1968, Kershner proved that there are anisohedral shapes for convex shapes. He found types of convex pentagonal tiling and claimed that there were a total of eight pentagonal tiles, but he would be proven wrong later be Richard James who found and introduced a ninth type and so on. Nowadays we know fifteen types of convex pentagon tiles.

In 2005, Rhoads proves that anisohedral shapes also exist for polyominoes.

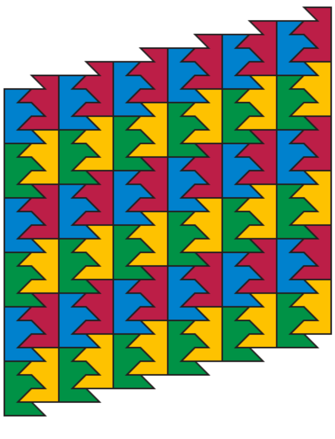

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Figure 1: Tiling example of the plane by Heesch.

3 Shapes

A polyomino is a geometric figure that is formed by one or more equal squares, edge to edge. There a infinite many shapes but so far there is no algorithm to decide if 5 polyominoes have a plane tiling. There are some algorithms that decide whether convex shapes have plane tiling for quads, triangles and hexagons but it is impossible for $>=$ 7 sized shapes. In the case of pentagons, it is unknown if an algorithm of this kind exist.

4 Tiling

4.1 Isohedral Shape

Isohedral shapes are figures that are repeated regularly over a plane and where you can shift or rotate the shape inside the plane and the tiling will look the same. 2

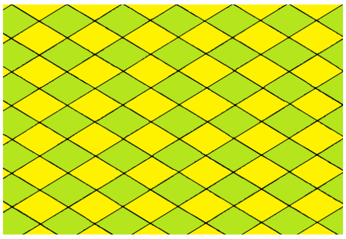

Figure 2: This is an example of an Isohedral.

4.2 Anisohedral

There are also tilings shapes where there is no isohedral found in the tiling plane.

4.3 K-Isohedral Shapes

K-Isohedral shapes are have a k-isohedral tiling but no c-isohedral tilings for any $c < k$. When ever K is greater than 6 there are still no examples to be found.

5 Plane

5.1 Finite regions

Input: polyominoes P(replaces plane) and T(usually fixed).
Output: whether P can be tiled by T.

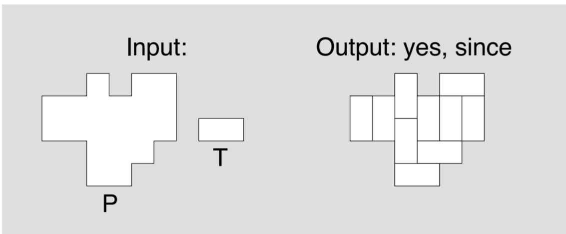

Figure 3: Input example

6 Tiling in 3D

In 2011, Taylor Socolar proved that an aperiodic polyhedron exists, even though there are no 2D aperiodic shapes known. It remains an open question whether there exist a polycube. A year later Vuillon Gambini proved that for every k, there exist a k-anisohedral polycube.

7 Surrounding

The study of surroundings is similar to plane tilings. Surroundings involve the study of algorithms to determine if a given polyomino can be surrounded by copies of this polyomino. Dr. Winslow introduced us to a game called "Good Fences" which is about finding surroundings for different polyominoes with different difficulties.

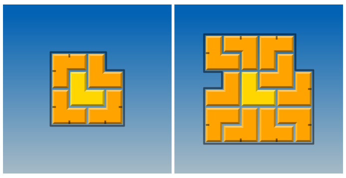

Figure 4: Example of two possible surroundings for the same polyomino. (Screenshot taken from Good Fences)