

Constraint Logic

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Constraint Logic

Can be viewed as a Model of Computation

QBF (Quantified Boolean Formulas) has no geometric properties

Definition: A constraint graph is a directed graph with edge weights $\in \{1, 2\}$ called red or blue, respectively
The inflow at each vertex is the sum of the weight on inward-directed edges
Each vertex has a non-negative minimum inflow these are the constraints

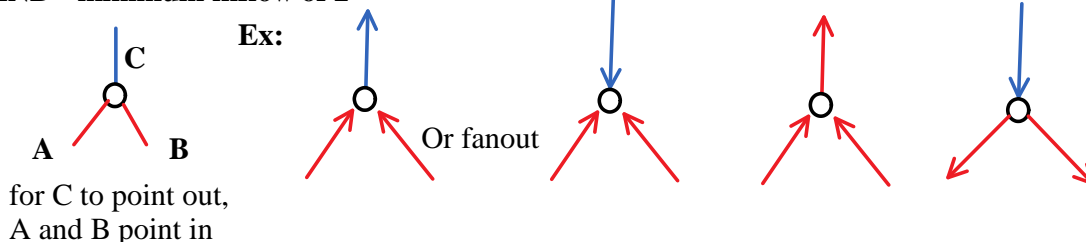
Definition: A legal move is the reversal of a single-edge orientation, resulting in another legal configuration

Games on a CG(Constraint Graph)

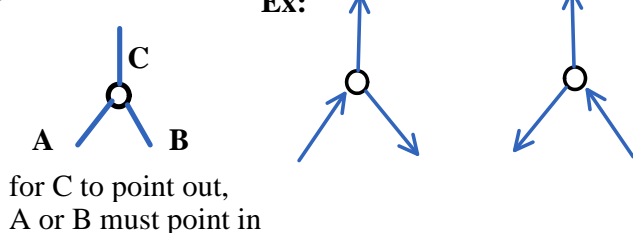
- Single-player puzzle – the goal is to reverse a single edge
- Multiplayer game – each edge may be controlled by a player
- Deterministic Games – a unique sequence of reversals is forced
- Bounded games – each edge may flip only once

Gadgets

AND - minimum inflow of 2



OR



We can't build a "not" gate

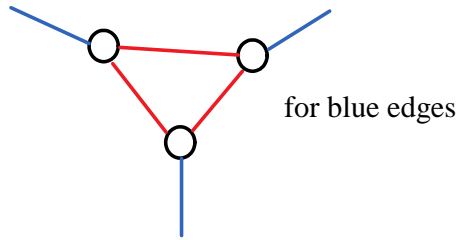
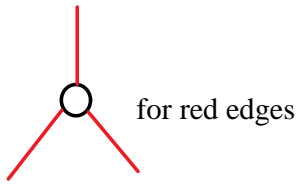
Logic is monotone

- Monotone boolean functions \Rightarrow AND/OR
 - Called monotone because when variable changes from false to true, the formula can never change to false
- Sufficient for computation

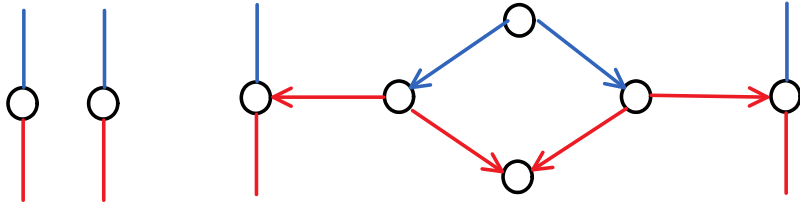
Planar Constraint Graphs

Bounded deterministic C.L. is the only one that isn't equivalent with planarity(?)

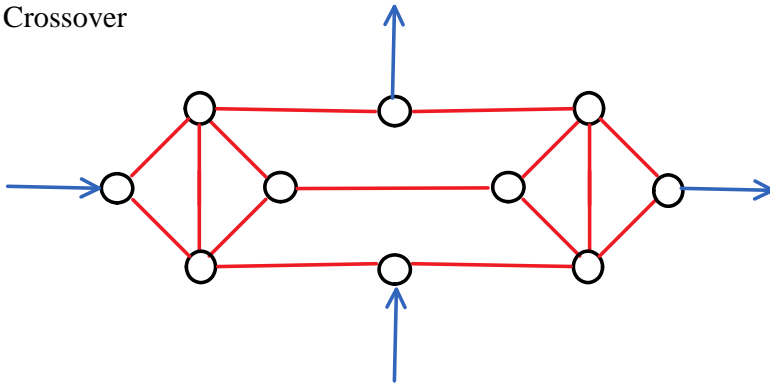
Choice Vertex



Connecting Different Edge Types



Crossover



Basic Problem: Nondeterministic Constraint Logic

Instance: Constraint graph G , and edge e in G

Question: Is there a sequence of moves on G that eventually reverse e ?

Zero-Player Games (Simulations)

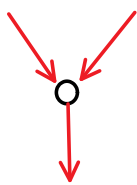
Cellular Automata

One-Player Games

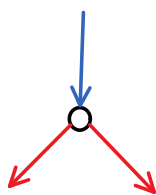
Bounded Nondeterministic C.L. (Bounded NCL)

- NCL only allowed to flip each edge once

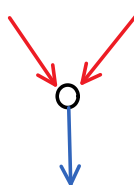
Theorem: Bounded NCL is NP-complete



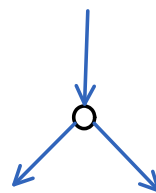
Choice



AND



Fanout



OR



Free Edge

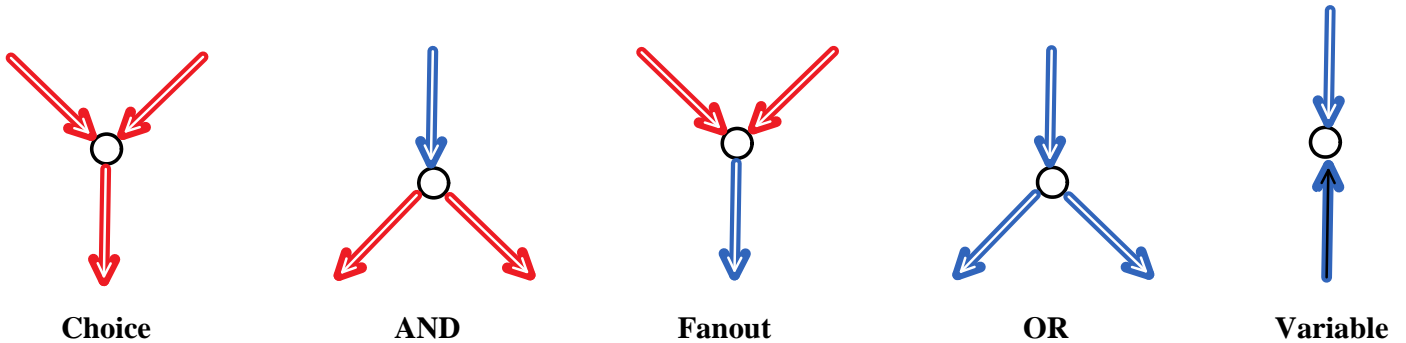
Protected OR – two of it's edges cannot simultaneously be directed inward

Two-Player Games

Bounded Games

- Resources that's used up
- Bounded two-player C.L. (Bounded 2CL)
- Instance: Constraint graph G, partition of the edges of G into sets B and W and edges $e_b \in B$ and $e_w \in W$
- Questions: Does white have a force win in the game?
 - Players W and B alternate making moves on G

Theorem: Bounded 2CL is PSPACE-complete



Unbounded Games

Two-Player Constant Logic (2CL)

Instance AND/OR constant graph... etc

TheoremL 2CL is EXPTIME-complete

Summarize the reduction

TipOver

Example for one-player game; unbounded

- To get to the red square, tip over to walk over to win

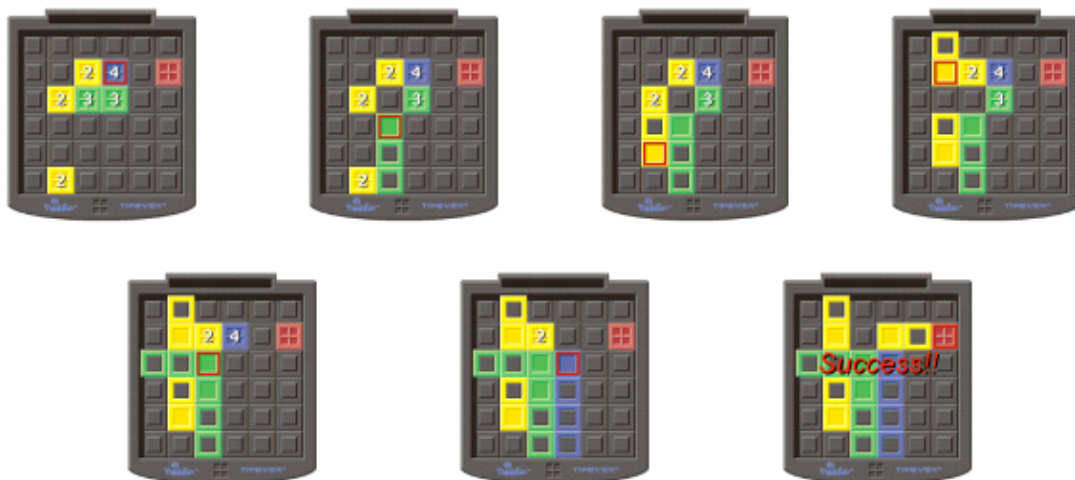


Figure 9-2: A sample TipOver puzzle and its solution.

We're given a constraint graph and given AND/OR/Fanout. We need to a wirte to go to left or right; size 2

- a directed edge in the graph
- AND: 2 different paths is needed; go back later and visit the
- OR: 1 path of 2
- don't assume player goes back

Sliding-Coin Puzzles

- Coins or vertex and at least two steps apart
- Is reversible
- Show reversible AND/OR's gadget to get PSPACE-complete

Plank Puzzle

- Need to make AND/OR gadget
- OR: has two options to go to output
- AND: going back to the path

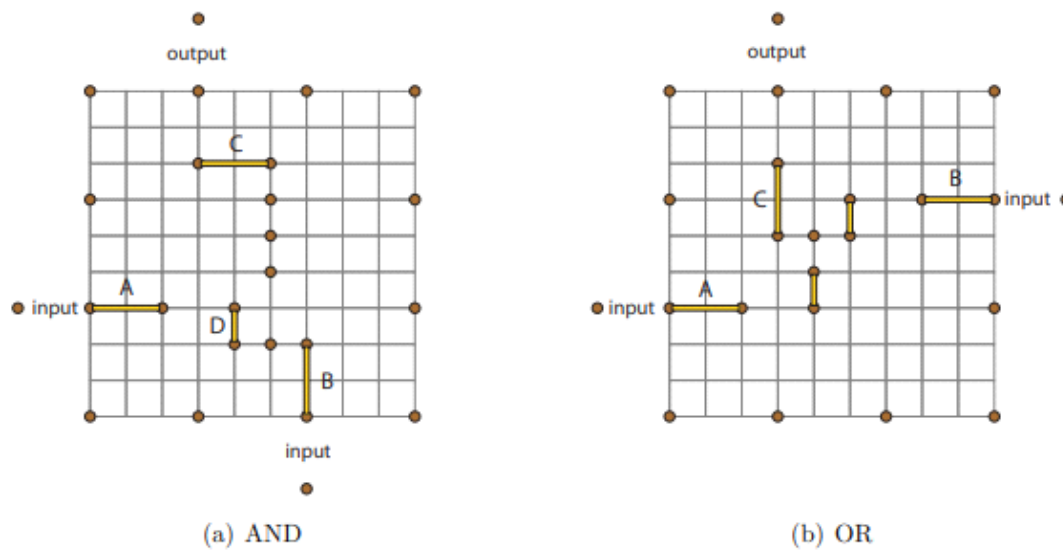


Figure 9-14: Plank-puzzle AND and OR vertices.