Group R Notes

Class groups chose a game to present a reduction. Some of the games that were available to choose from were Portal, Super Mario, Hitori, Angry Birds and many others.

The two most interesting and used reductions were ***3-Satisfiability*** and ***3-Partition*** in which will be briefly discussed in these notes.

Note – These reductions have much more intricate details and background in which the student is encouraged to research on their own if they are interested.

The Satisfiability Problem

The study of Boolean expressions generally is concerned with the set of truth assignments in which these assignments are set to either 0 or 1. These Boolean expressions can result to either true or false.

NP-Completeness needs only a simpler question: Does there exist a truth assignment making the expression true?

A special case of the satisfiability problem (SAT) is where all formulas are in conjunctive normal form (CNF). Conjunctive Normal Form is a conjunction of one or more clauses, where a clause is a disjunction of literals. You can think of it as an AND of ORs.

In **3SAT** we restrict the satisfiability problem to only allow three literals per clause. We are allowed to have infinite many clauses as well as literals but only three literals per clause.

For example:

We have *n* literals where each one has an assignment to either true or false. We will use the symbol ¬ to show negation:

$$(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee \neg x_4)$$

Optimization Problem: What is an assignment to the variables to satisfy the entire clause? In other words, what satisfies an entire clause? A clause must contain at least one literal that is assigned to true.

Input: A collection of clauses *C* where each clause contains exactly three literals over a set of Boolean variables *v*.

Output: Is there a truth assignment to *v* such that each clause is satisfied?

Let's assign values to our literals shown in the expression above.

$$x_1 = T\ ,\ x_2 = F\ ,\ x_3 = T\ ,\ x_4 = F\ ,$$

We can rewrite this as (T or F or T) AND (F or F or T) in which results to **true** thus, resulting in a satisfied 3SAT formula.

Three Partition

This problem is to decide whether a given multiset of integers can be partitioned into triples that all have the same sum. More precisely, given a multiset S of $n = 3, m$ positive integers, can S be partitioned into m triplets $S_1, S_2, \ldots, S_m$ such that the sum of the numbers in each subset is equal?

What is a multiset? It is a modification of the concept of a set that, unlike a set, allows for multiple instances for each of its elements.

The subsets $S_1, S_2, \ldots, S_m$ must form a partition of S in the sense that they are disjoint, and they cover S.

For example:

The set {20, 23, 25, 49, 45, 27, 40, 22, 19} can be partitioned into three sets: {20, 25,45}, {23, 27, 40}, and {49, 22, 19} each of which sum to 90.