

Def'n. A combinatorial game is a 2-player game played between Louise (Left) and Richard (Right). The game consists of the following:

1. A set of possible positions. (states of the game)
2. A move rule indicating for each position what positions Louise can move to and what ones Richard can move to.
3. A win rule indicating a set of terminal positions where the game ends.

Each terminal position has an associated outcome

- either L wins and R loses (+ -)
- or R wins and L loses (- +)
- or it's a draw (00)

To play - choose starting position, and designate player to go first
 - take turns until terminal position.

Normal play games - win rule is last player to move wins.
 - first person w/o move loses.

Misère play - last person to play loses.

Game Trees

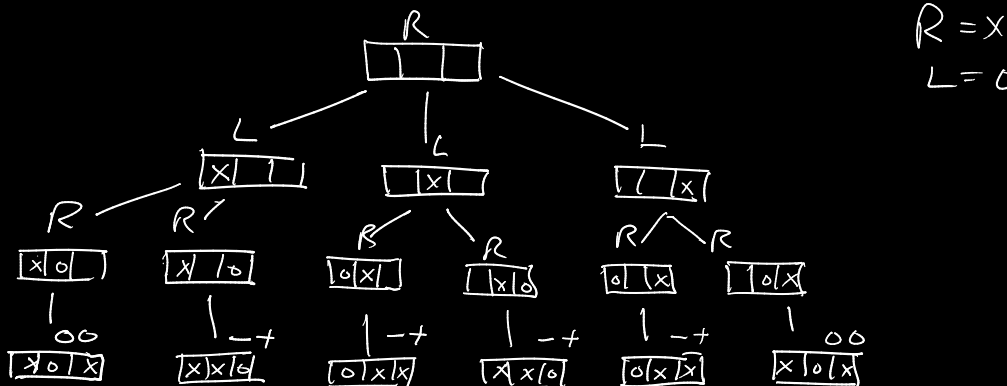
Zermelo's Thm.

• Tic - TTT on a 1x3 board

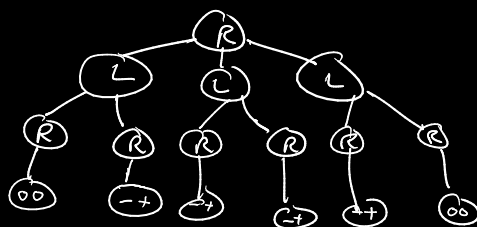


2 adjacent symbols
 is a win

• each branch is a choice. leaves are outcomes



WLD tree



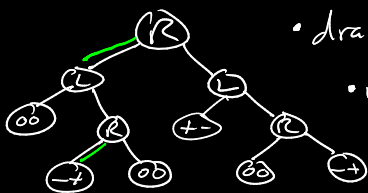
• unified way to think about combinatorial game play

• Some trees are unbounded
 • chess

• limit ourselves to finite trees.

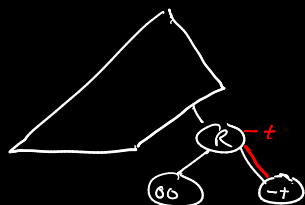
Strategy

- formalize a plan to win
- a set of decisions indicating which move to make at each node.
 - winning - best strategy - guarantees win
 - drawing strategy - guarantees a player doesn't lose
 - rational play - a player plays to win.



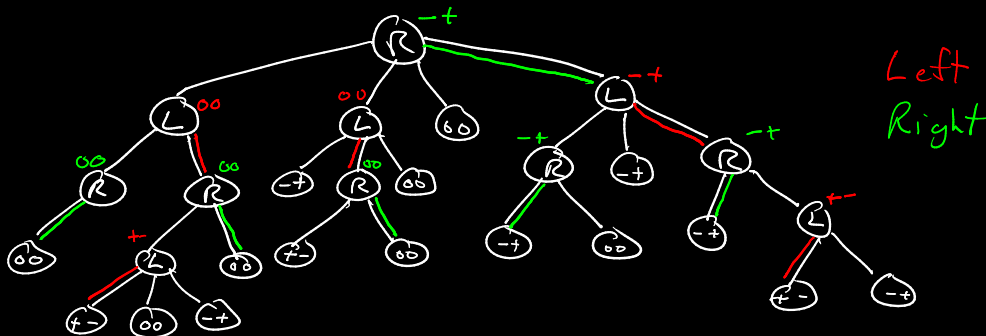
Working Backwards

- considering entire game tree, it may be hard to decide a move at the top, but not at the bottom.



since we know the decision at this node, we can think of it as terminal and mark it as \rightarrow

Procedure: A player has a decision to make at node N . We know outcome under rational play for all nodes from N . Choose best possible outcome for this player. Indicate on graph and mark N w/ outcome. Continue to root.



under rational play - Richard w/ always win b/c the root is marked \rightarrow

Zermelo's Thm.

Every WLD tree is one of

Type	Description
$+-$	L has a winning strategy
$-+$	R has a winning strategy
00	Both players have draw strategies

Some games

PUB - pick up bricks

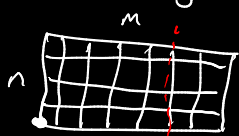


- each player can pick up 1 or 2 bricks.



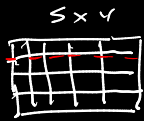
- 2nd player win

Chop - $m \times n$ grid



7×4

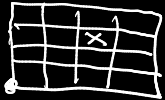
• make horizontal or vertical cut



5×4

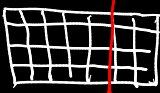
5×3

Champ - $m \times n$ grid



pick a piece and remove everything above and to the right

Cut-cake - $m \times n$ grid



- L only makes vertical cuts
- R only makes horizontal cuts

