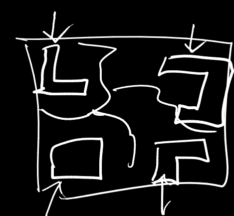
Prop. If α and β are both type L (R), then $\alpha+\beta$ is type L (R).

Types of sums

+	L	R	N	P
L	L	?	?	L
R	?	R	?	R
N	?	?	?	N
P	L	R	N	P

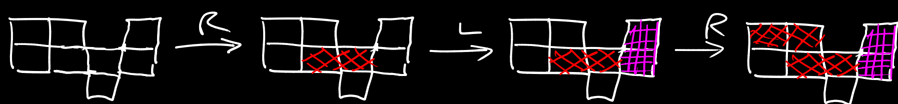
? can not be answered in general
game / position specific.

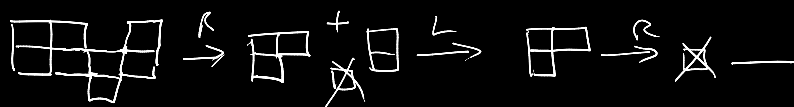


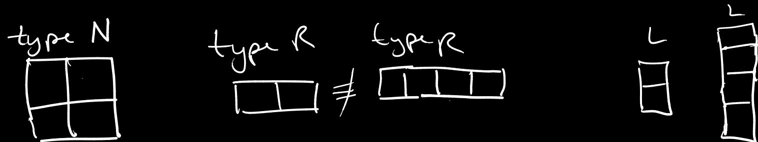
Indeterminate sums

Domineering

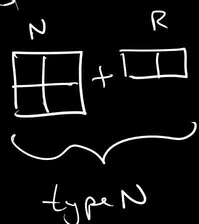
Assume R is horizontal, L is vertical



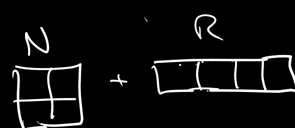




Sums



$N+R \Rightarrow N$

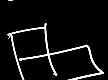


$N+R \Rightarrow R$

Equivalence (abstract algebra)

- Move beyond investigating one game at a time

Dom.



α

PUB

α'

- Player N wins in either game.
- If we take any other game position β, then $\alpha+\beta \equiv \alpha'+\beta$ because they play the same.

Def'n: Two positions α and α' in normal-play games are equivalent if $\forall$ position β in any normal-play game, $\alpha+\beta$ and $\alpha'+\beta$ have the same type

Notation $\alpha \equiv \alpha'$

Equivalence Relations

If α, β, γ are positions in normal-play games.

- $\alpha \equiv \alpha$ (reflexivity)
- $\alpha \equiv \beta$ implies $\beta \equiv \alpha$ (symmetry)
- $\alpha \equiv \beta$ and $\beta \equiv \gamma \rightarrow \alpha \equiv \gamma$ (transitivity)

can determine type by finding equivalent position.

$\Rightarrow$ Equivalent positions have the same type, but not all positions of the same type are equivalent.

Algebra w/ $\equiv$, $+$

If α, β, γ are pos. in normal-play games

- $\alpha + \beta \equiv \beta + \alpha$ (commutativity)
- $(\alpha+\beta)+\gamma \equiv \alpha + (\beta+\gamma)$ (associativity)

Lemma. Given positions α, β in normal-play games

1. If $\alpha \equiv \alpha'$, then $\alpha + \beta \equiv \alpha' + \beta$
2. If $\alpha_i \equiv \alpha_i'$ for $1 \le i \le n$, then $\alpha_1 + \dots + \alpha_n \equiv \alpha_1' + \dots + \alpha_n'$
3. If $\alpha_i \equiv \alpha_i'$ for $1 \le i \le m$ and $\beta_j \equiv \beta_j'$ for $1 \le j \le n$, then
$$\{\alpha_1, \dots, \alpha_m \mid \beta_1, \dots, \beta_n\} \equiv \{\alpha_1', \dots, \alpha_m' \mid \beta_1', \dots, \beta_n'\}.$$

Lemma. If β is type P, then $\alpha + \beta \equiv \alpha$ (type).

For normal play games, type P behaves like 0 for addition $6+0=6$.

Cor. If α and α' are type P, then $\alpha \equiv \alpha'$.

Cor. If $\alpha + \beta$ and $\alpha' + \beta$ are type P, then $\alpha \equiv \alpha'$