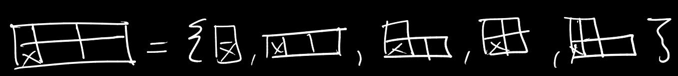
$\boxtimes = \{\} \equiv *0$

$\vdots$

Chomp

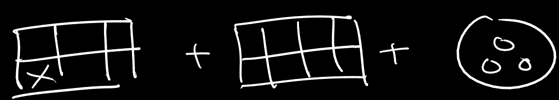
By mex

 $\equiv \{*0, *1, *2, *3\} \equiv *4$

Chop

 $\equiv \{*0, *1, *3\} \equiv *2$

Finding equiv. sums



chomp chop Nim

$*4 \quad + \quad *2 \quad + \quad *3 \equiv *(4 \oplus 2 \oplus 3) \equiv *5$

How do we play - balancing

$*4 + *2 + *3 \equiv *4 + *2 + *(2+1)$

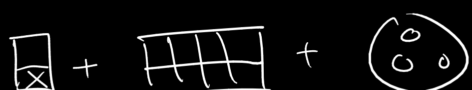
$2^2 + 2^1 + (2^1 + 2^0) \longrightarrow$ pick largest odd 2^i component

$2^2 + 2^0$

reduce component # so that all powers of 2 are even

$2^0 + 2^1 + (2^1 + 2^0)$

need to turn $*4 \longrightarrow *1$



chomp chop Nim

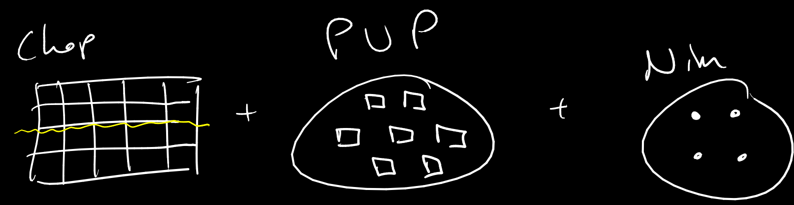
$*1 + *2 + *3$

type P $\equiv *0$

PUB Thm. Let $n = 3l + k$ where $0 \le k \le 2$. Then a PUB position of n bricks is equiv. to $*k$. always $*0, *1, *2$

Chop Thm. For every $m, n \ge 1$, an $m \times n$ position in Chop is equivalent to $*(m-1) + *(n-1)$. 1×1 is terminal

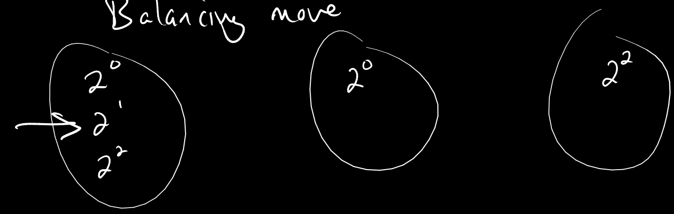
Nimber equivalent



$*3+*4 \equiv *7$ + $*1$ + $*4$

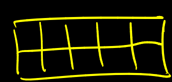
Nimber equiv. $*(7\oplus 1\oplus 4) \equiv *((\cancel{1}+2+\cancel{4})\oplus \cancel{1}\oplus \cancel{4}) \equiv *2$

Balancing move



largest odd 2^i

By moving the chop position to be of value $*5$, we balance the games. from $*7 \to *5$

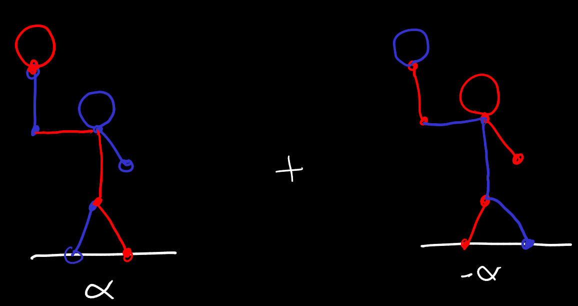


$\equiv *5 \equiv *1 + *4$

Partizan Games

- All normal-play impartial games are equiv. to a nimber.
- $\Rightarrow$ find a similar thing for partizan games.

Hackenbush - normal play game between L, R. The game board is a graph w/ colored edges, some are attached to the ground. R cuts blue, L cuts red. Anything w/o a path to ground is removed.



Def'n: $\cdot 0$ is the Hb position w/ no edges.
$\cdot 0$ is type P.

Prop. A Hb position $\alpha \equiv \cdot 0$ iff α is type P.

The sum operator $\alpha + \cdot 0 \equiv \alpha$ for all normal play games.

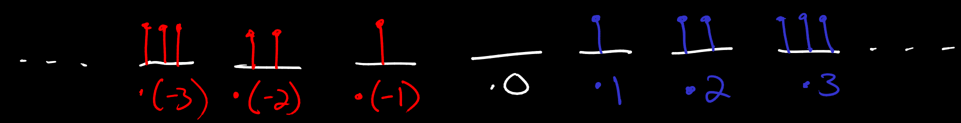
Negation. Reversing the colors, reverses the roles of the players. switching all colors is negation

Prop. If α and β are Hb positions, then

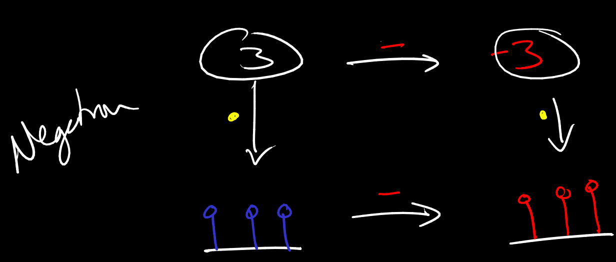
1. $-(-\alpha) \equiv \alpha$
2. $\alpha + (-\alpha) \equiv \cdot 0$
3. $\beta + (-\alpha) \equiv \cdot 0$ implies $\alpha \equiv \beta$

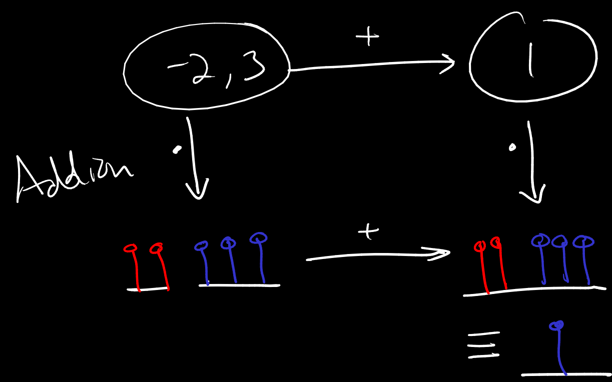
Integer positions

For every pos. int. n, define $\cdot n$ to be the Hb position consisting of n isolated black (blue) edges



$\cdot(-\alpha) \equiv -(\cdot \alpha)$





Thm. For any integers.

1. $-(\cdot n) \equiv \cdot(-n)$
2. $(\cdot m) + (\cdot n) \equiv \cdot(m+n)$

$\cdot n > 0$ R has advantage

$\cdot n < 0$ L has advantage