CSCI 4341
Assignment 3 (100 points)

Standard things apply about being due and such.

1. (9 pts) For each game below, construct a game tree with Louise moving first from the indicated starting position.
   (a) Tic starting from a blank board.
   (b) Pick-Up-Bricks starting with 4 bricks.
   (c) Chop starting from a 2 x 3 board.

2. (9 pts) In each game from Problem 1, show the WLD tree and a winning or drawing strategy for each player (6 strategies).

3. (9 pts) Use the $\gamma = \{\alpha_1, \ldots, \alpha_m \mid \beta_1, \ldots, \beta_n\}$ notation to represent the following positions:
   (a) a 2 x 3 position in Chop
   (b) a 4 x 5 position in Cut-Cake
   (c) a blank 2 x 3 board in Domineering

4. (10 pts) Use the $\gamma = \{\alpha_1, \ldots, \alpha_m \mid \beta_1, \ldots, \beta_n\}$ notation to represent the following positions:
   (a) a 3 x 3 piece in Cut-Cake plus 4 bricks in Pick-Up-Bricks
   (b) a blank 2 x 3 board in Domineering plus 5 bricks in Pick-Up-Bricks

5. (10 pts) Use the proposition (table for determining NPRL) to find the types of positions below:
   (a) a 2 x 4 position in Cut-Cake
   (b) a blank 2 x 3 board in Domineering

6. (10 pts) Prove that each of the following positions is type N by finding a winning strategy for each player when he or she moves first. Don't construct the game tree, but instead indicate the first move(s) and explain how to proceed from there.
   (a) The sum of a blank 2 x 2 board in Domineering and 2 bricks in Pick-Up-Bricks.
   (b) The sum of a 3 x 3 piece in Cut-Cake and 5 bricks in Pick-Up-Bricks.

7. (10 pts) Using the definition of $\equiv$ indicate for each of the following whether $\alpha \equiv \alpha'$ or $\alpha \not\equiv \alpha'$ or if the provided information is inconclusive. Use the Lemma stating if $\alpha + \beta$ and $\alpha' + \beta$ are both type P, then $\alpha \equiv \alpha'$.
   (a) $\alpha + \beta$ and $\alpha' + \beta$ are both type N.
   (b) $\alpha + \beta$ is type N and $\alpha' + \beta$ is type L.
   (c) $\alpha + \beta$ and $\alpha' + \beta$ are both type P.
   (d) $\alpha + \beta$ is type L and $\alpha' + \beta$ are is type R.
   (a) $\alpha + \beta$ and $\alpha' + \beta$ are both type R.

8. (9 pts) Consider the two Domineering positions: α  , β 
   (a) Show that both α and β are type N by finding the first move of a winning strategy for the first player and giving a brief explanation of how to play from there.
   (b) Find the types of $\alpha + \alpha$ and $\alpha + \beta$ and provide a short proof in each case.
   (c) Show that $\alpha \not\equiv \beta$.

9. (24 pts) Find Domineering positions α and β of the indicated types that sum to a position $\alpha + \beta$ of the indicated type:

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	(l)
α	L	L	R	R	L	L	L	L	N	N	N	N
β	N	N	N	N	R	R	R	R	N	N	N	N
$\alpha + \beta$	L	N	R	N	L	R	N	P	L	R	N	P