CSCI 4341

Assignment 4 (200 points)

Standard things apply about being due and such.

1. (12 pts) Find the binary expansion for the following integers:
- (a) 23
- (b) 47
- (c) 163

2. (18 pts) Evaluate:
- (a) $7 \oplus 4 \oplus 3$
- (b) $14 \oplus 24 \oplus 32$
- (c) $19 \oplus 13 \oplus 23 \oplus 57$

3. (10 pts) Find a nimber which is equivalent to a 2×6 position in Chop.

4. (18 pts) Use the balancing procedure to find a winning move in each of the following Nim positions:
- (a) $*3 + *4 + *5$
- (b) $*7 + *9 + *14 + *6$
- (c) $*19 + *37 + *28 + *33$

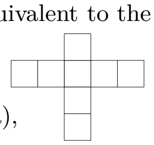
5. (36 pts) For each position, find an equivalent nimber and a winning move if it exists:
- (a) a 2×3 array in Chomp plus a 2×4 array in Chop plus a Nim stack $*5$
- (b) a 4-brick position in Pick-Up-Bricks plus a 5×3 array in Chop plus a Nim stack $*7$
- (c) an 11-brick position in Pick-Up-Bricks plus an 18×24 array in Chop plus a Nim stack $*20$

6. (12 pts) Find the binary expansion for each of the following fractions:
- (a) 15/16
- (b) 61/32
- (c) 317/128

7. (18 pts) Draw each of the given dyadic positions in Hackenbush:
- (a) •(5/8)
- (b) •(23/32)
- (c) •(-121/64)

8. (10 pts) Which dyadic numbers are born on days 4 and 5?

9. (42 pts) Find a dyadic position equivalent to the following given positions:
- (a) A 3×6 board in Cut-Cake.
- (b) The Domineering position
- (c) The sum of the position in (a), (b), and •(-5/4).



10. (24 pts) For a position γ in a partizan game, a *winning move* for Louise is any move to a position of type **L** or **P**, while a *winning move* for Richard is any move to a position of type **R** or **P**. Both have a winning strategy playing second for the resulting position. Let $a_1, \ldots, a_n$ be dyadic numbers, and consider the position $\alpha = \bullet a_1 + \bullet a_2 + \ldots + \bullet a_n$.
- (a) If $a_1 + a_2 + \ldots + a_n \geqslant 1$, what are Louise's winning moves from α?
- (b) If $0 < a_1 + a_2 + \ldots + a_n < 1$, what are Louise's winning moves from α?

Bonus: (5 pts) For every positive integer n, find a dyadic number a_n so that a $3 \times n$ position in Cut-Cake is equivalent to $\bullet a_n$. Prove your formula holds.

Bonus: (5 pts) Given a hollow 3D cube. How many ways could you unfold the surface to make a connected shape in 2D. Can you tile the plane with all unfoldings?