

## CSCI 4325

### Review 3

**Problem 1:** For this problem use the TM  $M_1$ , which appears in Example 3.9 on page 173. For each part, give the sequence of configurations that  $M_1$  enters when started on the input string ending with a reject or accept.

- (a) 0#0
- (b) 1##1
- (c) 10#11
- (d) 10#10

**Problem 2:** Give a state transition diagram for the Turing Machine  $M_3$  specified in Example 3.11 which decides the language  $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$ .

**Problem 3:** Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet  $\Sigma = \{a, b\}$ .

- (a)  $\{w \mid w \text{ contains twice as many } a\text{'s as } b\text{'s}\}$
- (b)  $\{w \mid w \text{ does not contain twice as many } a\text{'s as } b\text{'s}\}$

**Problem 4:** Which of the following problems about Turing machines are solvable, and which are undecidable? Explain your answers carefully.

- (a) To determine, given a Turing machine  $M$ , a state  $q$ , and a string  $w$ , whether  $M$  ever reaches state  $q$  when started with input  $w$  from its initial state.
- (b) To determine, given a Turing machine  $M$  and a symbol  $a$ , whether  $M$  ever writes the symbol  $a$  when started with the empty tape.

**Problem 5:** Give a formal Turing Machine that sorts a bunch of 1s, 2s, and 3s when given a string of them in any order. Example input: 223123213231211221333131 output: 1111111222222233333333. This should be done with the tape alphabet being only the input alphabet and the blank symbol.

**Problem 6:** Give a formal Turing Machine that when given barrels of apples and oranges will decide which one has more. Example input: [AAAA]\_[OOO]. The underscore is a blank cell. In the space in the middle it should place either  $>$ ,  $=$ , or  $<$ . Example output: [AAAA] $>$ [OOO]. The tape alphabet is  $[, ], A, O, >, <, =, \_$ .