

CSCI 4325

Review 4

Problem 1: Let $\text{RELPRIME} = \{\langle x, y \rangle \mid x \text{ and } y \text{ are positive integers that are relatively prime}\}$. Given the following algorithm to test if two positive integers are relatively prime, let n be the maximum number of decimal digits in x and y . Analyze the running time of this algorithm, using O -notation. Explain the details and give your reasoning for each step.

On input $\langle x, y \rangle$ where x and y are positive integers.

1. Repeat until $y = 0$:
2. Assign $x \leftarrow x \bmod y$.
3. Swap x and y .
4. Output x . If the result is 1, *accept*; otherwise *reject*.

Problem 2: Show that the Traveling Salesman Problem is **NP**-complete.

1. Show that the problem is in **NP**.
2. Show that *Undirected Hamiltonian Path* is poly-time reducible to the problem.

Problem 3: Prove that bin packing is **NP**-complete.

1. Show that the problem is in **NP**.
2. Show that the *Partition* problem is poly-time reducible to the problem.

Problem 4: Explain why problems like the Partition problem are only considered *weakly NP*-hard. What is pseudopolynomial time?

Problem 5: REACHABILITY is the following problem: given a directed graph G and two nodes s and t of G , decide whether there is a path from s to t . Give a polynomial time algorithm for this problem.

Problem 6: Show that *2SAT* is in **NL** by giving an algorithm to solve it using nondeterministic log-space.