**CSCI 4325**
**Practice for Quiz/Exam 1**

**Problem 1:** What is the minimum number of shots needed to guarantee hitting a battleship (a 4 × 1 rectangle) on a 10 × 10 board? The battleship can be located anywhere on the board and may be oriented either horizontally or vertically. You may assume that there are no other ships. A "shot" is a blind guess of a square on the board.

**Problem 2:** Let $c_1 x^n + c_2 x^{n-1} + \ldots + c_n x + c_{n+1}$ be a polynomial with a root at $x = x_0$. Let $c_{max} = max_{1 \le i \le n+1} |c_i|$. Prove that

$$|x_0| < (n+1)\frac{c_{max}}{|c_1|}.$$

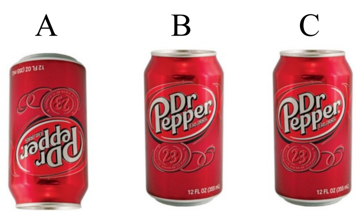
**Problem 3:** Give state diagrams of DFAs recognizing the following languages. The alphabet is $\Sigma = \{0,1\}$.

(a) $L_1$ = {$w$ | every odd position of $w$ is a 1}
(b) $L_2$ = {$w$ | $w$ contains an odd number of 0's, or exactly two 1's}
(c) $L_3$ = {$w$ | $w$ contains the substring "000"}
(d) $L_4$ = {$w$ | $w$ ends with "101"}

**Problem 4:** For each binary relation $f \subseteq A \times B$, draw the mapping and say if it is injective, surjective, bijective, neither, or not a function.

(a) $f$ = {(1, a), (2, b), (3, a), (4, c)}, A={1, 2, 3, 4}, B ={a, b, c}
(b) $f$ = {(1, a), (2, b), (3, d), (4, c)}, A={1, 2, 3, 4}, B ={a, b, c, d}
(c) $f$ = {(1, a), (2, b), (3, d), (4, c)}, A={1, 2, 3, 4}, B ={a, b, c, d, e}
(d) $f$ = {(1, a), (2, b), (3, a), (4, c)}, A={1, 2, 3, 4}, B ={a, b, c, d}
(e) $f$ = {(1, a), (2, b), (3, d), (1, c)}, A={1, 2, 3}, B ={a, b, c}
(f) $f$ = {(1, a), (2, b), (3, d)}, A={1, 2, 3, 4}, B ={a, b, c}

**Problem 5:** Suppose there are three cans of soda (obviously Dr. Pepper, otherwise just throw them in the trash), which we'll label A, B, and C. You notice that can A is upside down. If you are required to flip two cans at a time, can you get all the cans facing right-side up in no more than 6 moves? If yes, give the sequence. If not, prove that it can't be done. Make a DFA to support your answer.




**Problem 6:** Draw the following graphs.

(a) $G_1$ = (V, E) where V={a, b, c, d, e} and E={{a, b}, {a, c}, {d, a}, {b, c}, {c, d}}.
(b) $G_2$ = (V, E) where V={a, b, c, d, e} and E={(a, b), (a, c), (d, a), (b, c), (c, d), (d, c)}.