CSCI 4325
Practice for Quiz/Exam 4

Problem 1: Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\Sigma = \{a, b\}$.

(a) $\{w \mid w$ contains twice as many a's as b's$\}$
(b) $\{w \mid w$ does not contain twice as many a's as b's$\}$

Problem 2: Give a state transition diagram for the Turing Machine M_3 specified in Example 3.11, which decides the language $C = \{a^i b^j c^k \mid i \times j = k$ and $i, j, k \geq 1\}$.

Problem 3: Which of the following problems about Turing machines are solvable, and which are undecidable? Explain your answers carefully.

(a) To determine, given a Turing machine M, a state q, and a string w, whether M ever reaches state q when started with input w from its initial state.
(b) To determine, given a Turing machine M and a symbol a, whether M ever writes the symbol a when started with the empty tape.

Problem 4: Give a formal Turing Machine that sorts a bunch of 1s, 2s, and 3s when given a string of them in any order. Example input: 22312321323121122133313 1 output: 1111111122222222333333 3. This should be done with the tape alphabet being only the input alphabet and the blank symbol.

Problem 5: Give a formal Turing Machine that when given barrels of apples and oranges will decide which one has more. Example input: [AAAA]_[OOO]. The underscore is a blank cell. In the space in the middle it should place either >, =, or <. Example output: [AAAA]>[OOO]. The tape alphabet is {[,],A,O,>,<,=,_}.

Problem 6: Prove the following problems are decidable.

(a) Hampath = $\{\langle G, s, t\rangle$: Graph G has a Hamiltonian path from vertex s to vertex $t\}$.
(b) Vertex-Cover = $\{\langle G, k\rangle$: Graph G has a vertex cover of size $k\}$.
(c) Path = $\{\langle G, s, t\rangle$: Graph G has a path from vertex s to vertex $t\}$