CSCI 4325
Practice for Quiz/Exam 6

Problem 1: Let RELPRIME = {<*x, y*> | *x* and *y* are positive integers that are relatively prime}. Given the following algorithm to test if two positive integers are relatively prime, let *n* be the maximum number of decimal digits in *x* and *y*. Analyze the running time of this algorithm, using *O*-notation. Explain the details and give your reasoning for each step.

On input <*x, y*> where x and y are positive integers.

1. Repeat until $y = 0$:
2. Assign $x \leftarrow x \bmod y$.
3. Swap *x* and *y*.
4. Output *x*. If the result is 1, *accept*; otherwise *reject*.

Problem 2: Prove the following problems are in **P** by giving an algorithm and showing their run-times.

(a) Path = {<*G, s, t*>: Graph *G* has a path from vertex *s* to vertex *t*}.
(b) No-Path = {<*G, s, t*>: Graph *G* does not have a path from vertex *s* to vertex *t*}.
(c) Euler-Path = {<*G*>: Graph *G* has an Euler path}.
(d) Maximum-Matching = {<*G*>: A maximal matching for Graph *G*}.
(e) 1-pos-3-SAT = {<φ>: Boolean formula in 3-CNF form has a satisfying assignment and each clause has a variable that is not negated}.

Problem 3: Prove the following problems are in **NP** by giving a nondeterministic TM, and also by giving a deterministic verifier.

(a) Hampath = {<*G, s ,t*>: Graph *G* has a Hamiltonian path from vertex *s* to vertex *t*}.
(b) Vertex-Cover = {<*G, k*>: Graph *G* has a vertex cover of size *k*}.
(c) Traveling-Salesman-Problem, TSP = {<*G, k*>: Weighted graph *G* has a tour of length k}.
(d) Bin-Packing, BP = {<*S, c, k*>: *S* is a set of numbers that can be put into *k* bins of size *c*.}
(e) Minimum-Maximal-Matching, MMM = {<*G, k*>: Graph *G* has a maximal matching of size *k*}.
(f) 1-in-3-SAT = {<φ>: Boolean formula in 3-CNF form has a satisfying assignment and each clause only has one variable that is true}.

Problem 4: Given an algorithm that can solve a decision problem in O(t(n)) time (an oracle), show that we can also solve the optimization version with only a polynomial difference.