CSCI 4325
Practice for Quiz/Exam 7

Problem 1: Prove the following problems are in **L** by giving an algorithm and showing their run-times.

(a) Path = {<*G, s, t*>: Graph *G* has a path from vertex *s* to vertex *t*}.

(b) Majority = {<*w*>: the number of 1’s in the string is greater than the number of 0’s}.

(c) Proper-parentheses = {<*w*>: w is a string of properly nested parentheses}.

Problem 2: Show that *2SAT* is in **NL** by giving an algorithm to solve it using nondeterministic log-space.

Problem 3: Prove the following problems are **NP**-hard by giving a polynomial-time reduction from another NP-hard problem.

(a) HamCycle = {<*G*>: Graph *G* has a Hamiltonian cycle} (reduce from *Undirected Hamiltonian Path*)

(b) 2-m-SAT = {<φ, *k*>: φ is satisfiable with only *k* variables set to true} (reduce from *Vertex Cover*)

(c) Traveling-Salesman-Problem, TSP = {<*G, k*>: Weighted graph *G* has a tour of length k} (reduce from *Undirected Hamiltonian Path*)

(d) Bin-Packing, BP = {<*S, c, k*>: *S* is a set of numbers that can be put into *k* bins of size *c*} (reduce from the *Partition* Problem)

(e) Hitting-Set, HS = {<*S, k*>: *S* is a set of sets of numbers, and there is a set of size *k* that hits every set in *S*} (reduce from *Vertex Cover*)

(f) Longest-Path = {<*G, a, b, k*> | *G* contains a simple path of length at least *k* from *a* to *b*} (reduce from *Undirected Hamiltonian Path*)

Problem 4: Explain why problems like the Partition problem are only considered *weakly* **NP**-hard. What is pseudopolynomial time?